aracne

PAOLO CHRISTILLIN

GRAVITATION FOR THE SIMPLE MIND(ED)

aracne

aracne

ISBN
979–12–5994–705–5

FIRST EDITION
ROMA, MARCH 7TH 2022

In the memory of my parents to myself.
And to Galileo, Newton, Einstein and Occam.

Contents

Introduction

Prologue

De rerum natura, LUCRETIUS

We cannot but admire the ingenuity of a simple mind that led Thomas Lydiat to observe already in 1605 (i.e. before Kepler with whom he had a correspondence) that the earth was not orbiting the Sun on a circular orbit [1].

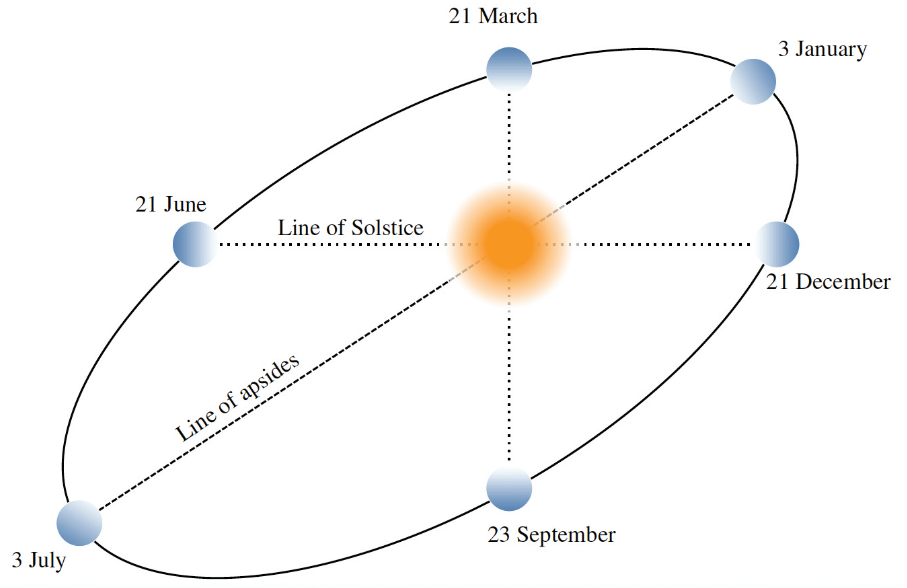

Figure 1. Between the autumn equinox (23 september) and the spring equinox (21 march) 179 days pass. In reverse 186. Thus asymmetry from a circular orbit is $(186-179)/(186+179) \simeq .02$ quite close to the ellipse eccentricity! This in spite of a one day uncertainty in the determination of the spring equinox.

Analogously an inquiring mind made Olbers [2] and others wonder why the same amount of light which reaches us at daytime does not come from the same stars at night, thus questioning the picture of an infinite static Universe.

This is reminiscent of one of the most fascinating Greek myths: that of the cave [3]. We live in a cavern where we try to reconstruct reality from the shadows which reach us from the outside and this indicates the close connection between cosmological speculations and terrestrial effects, i.e. the unity of the Universe [4].

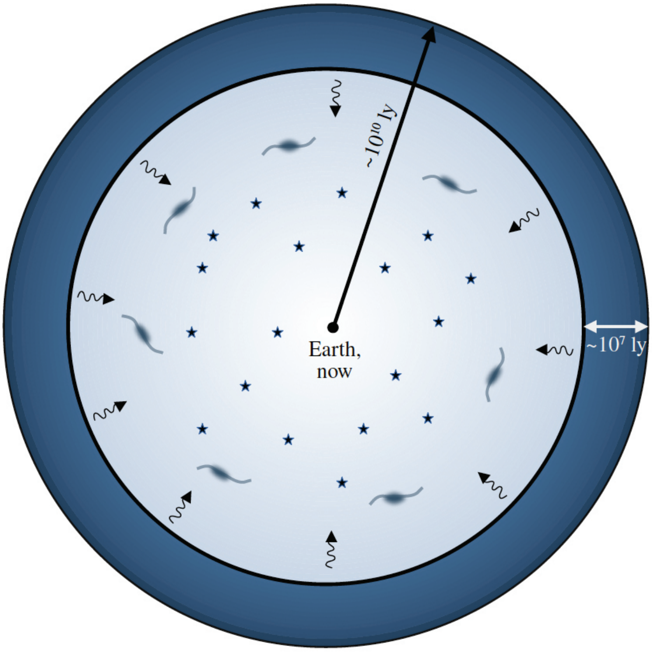

Figure 2. A sketch of our visible Universe, stars and galaxies (wavy lines). The farthest we look back is at $R' \simeq 1/1000\, R_U$ the Universe radius, at t' $\simeq 10^{14} s$ whose radiation still reaches us as a part of the background noise in old cathode ray tube tv sets. The $\simeq 10^7$ light years, according to the present reconstruction, have elapsed since the big bang. The time from the CMB to the present is about 12 billion years when structure formed, fact that might have justified a stationary Universe. So the greatest part of our past is visible, whereas the origin in spite of its very brilliant nature is invisible and just open to speculations. A complementary figure of the Universe evolution in chapter 4.

The modern Platonic cave is limited by the layer of the region of the cosmic microwave background (CMB) whose influence from the past reaches us at a temperature $T \simeq 2,7K$ with energy density

$E/V \simeq (kT)^4$. It was discovered by chance by Penzias and Wilson [5] in 1964 as a persistent background noise in a tv antenna. For that reason it is provocatively said that it might have been discovered before just with an ordinary tv set. Together with the Hubble-Lemaitre [6] relation it represents the major cosmological discovery of the last century. Its luminosity at night is of the order of 10^{-9} that of the sun at daytime. This solves the Olbers paradox as it will be discussed in the following. Having outlined qualitatively the limits of our treatment we then pass over to provide, in an as simple as possible way, a consistent parameter free formulation of gravitation accounting for the physics of the solar system and extending it to the cosmological scenario. Another sizable photon contribution exists: the higher energy later times CXB [7]. Its interpretation however does not lead to such fundamental information about the history of the Universe.

From Galileo to cosmology

«E caddi come corpo morto cade»
DANTE

The aim of this paragraph is two-fold. The first is to provide a short introduction motivating and highlighting the ingredients necessary for a correct treatment of gravitation i.e. essentially the extension of Newton's law, the second is more concerned with the examination and discussion of the unsettled problem of the present theoretical treatment.

Let us start with the General Relativity (GR) [8] approach and the Painleve' -Gullstrand (P-G) [9] [10] coordinates. The latter have, so to say, obtained validation as a solution of GR but have an independent support from a later more intuitive treatment [11] from which two important consequences can be gained. The first is the importance of the boundary conditions at infinity (whose use in cosmology is debatable), the second the uselessness of differential geometry for results which have been advertised as a fruit of this necessary approach. Indeed, contrary to Special Relativity (SR) a dependence on the metric is unescapable. The statements which are proper to one, do not hold true in another. Thus for instance whereas time and space dilation of

SR correspond to physical facts, similar statements in GR only have a meaning in a given metric. Physical results must of course be metric independent. The other advantage of the P-G metric is that it follows more closely the initial program of Einstein in the sense that it is explicitly based on the invariance of the velocity of light in all inertial frames.

The second point regards GR extension to cosmological problems. To make things understandable at a non rigorous level let us recall that it all starts by considering a spherical ball of given matter density and a mass m at its surface. Its motion is determined only by the matter inside so that its velocity v obeys the usual energy conservation equation

$$\frac{mv^2}{2} - \frac{GMm}{r} =?0 = \frac{v^2}{2} - G\rho' r^2$$

where ρ' stands for the usual clumsy $(4\pi/3)\rho$.

The question mark appears for two reasons. The first is that this equation is manifestly non relativistic (NR). The second is connected to the energy at ∞. Now an interesting remark is that by using in the previous equation Hubble-Lemaitre's s law

$$v = Hr$$

the dimensions of the ball disappear since both terms share the same r^2 *dependence.*

Therefore

$$H^2(t) - 2G\rho' =?0$$

can only be time dependent. Hence no constant is allowed in such a *homogeneous* equation[1] So homogeneity comes not as a postulate but just from a judicious consideration of the equation. Thus the radius r can be immediately identified with the Friedman-Robertson-Walker [12] scale factor (usually denoted confusingly by R) and here and henceforth by χ.

Of course the previous argument must be supplemented by some information about the matter density. It must reproduce some mean

1. Alternatively one could "predict" Hubble-Lemaitre's law from the assumption of the existence of a mean density and energy balance.

density, which seems to be the case in cosmology. In that case the result obtains. This is not a truism since one could define a (point like) density also in the case of the solar system. The equation would look formally the same. However in this case v/r would not be of course constant. *Thus the treatment of the discrete solar system case cannot be taken over to cosmology in spite of a formal similarity. We are in the presence of two distinct cases which cannot be described by the same equation*, and the claim that from Newtonian physics (even in the GR formalism) one could predict the Universe expansion is misleading.

Let us then come to relativistic extension of the previous equation. Clearly a direct substitution $v = c$ is untenable. Would we have any justification however for using nevertheless

$$c^2 - GM/r = 0 \tag{1}$$

leading to the well known black hole condition where $R_{b.h.} = GM/c^2$ is the half of the GR Schwarzschild radius R_S which should also represent the dimensions of a black hole?

Which of the two accounts for reality? The problem is not heuristic since the use of ρ' instead of $2\ \rho'$ suggests trivially that only half of the usual density enters thus making unnecessary to invoke the existence of dark energy, very reminiscent of ether.

If we consider

$$Mc^2 - GM^2/R = 0 = c^2 - GM/R$$

this can account for the possibility of bound objects to expand!

Indeed by increasing R, the potential term decreases so that its contribution to the energy increases and this has to be balanced only if M increases i.e. if matter is created.

Thus the previous relation treated as an equation Eq. 1 is what we take as describing the Universe evolution. This justifies the proposed extension of the NR equation of motion. Even if the results of the two approaches look very similar their theoretical interpretation is totally different. In that respect let us underline that is not by chance that GR reproduces the Michell-Laplace [13] [14] NR result. It just provides its formal back up. All of this will be detailed and formalized in the following.

Gravitation

It all starts with the famous Galilean law

$$z = z_0 - 1/2gt^2$$

where g is the acceleration at the earth surface $g \simeq 9,8m/s^2$ and z_0 the initial height. Where the related experiment was actually performed (leaning tower or inclined plane) is irrelevant to our purposes. The point is that this was the first quantitative result against the Aristotelian tradition. And this is also our first elementary approach to the discussion of gravitation. Shortly after came the revolutionary unification by Newton who described attraction by the earth on a body at its surface and on the moon in common terms with the famous law

$$F = ma = GMm/r^2$$

where G $\simeq 10^{-10}$ SI units and m the is traditionally the smaller mass whose motion is determined from the attraction by M.

It was only centuries after, with the final accommodation of electromagnetism (e.m.) by Maxwell, that it was realized that the formulation of this theory was approximate on fundamental grounds[2].

To see why this is so let us recall another fundamental contribution of Galileo, i.e. *the principle of relativity*.

No experiment can discover absolute motion. The well known experiment inside a ship is the first example of an ideal experiment (Gedankenexperiment) which will then play a fundamental role in SR.

A beautiful and elementary example of problems connected with causality for NR mechanics has been given by Fang Li Shi and Chu-Yao-Quan [16] and is reported in Fig 3).

The stroke of genius of Einstein was to extend the Galilean invariance to all physical phenomena (known at that time, except gravitation) i.e. also to electromagnetism. Indeed if one accepts that no

2. To correct a wide spread belief I report here an example from [15]. Consider two equal balls of different materials, one of lead and the other of cork say, with the same shape so that the friction force is the same for both. However they do not fall with the same acceleration since the mass is different and therefore the contribution to the *acceleration* of the friction force is inversely proportional to the mass.

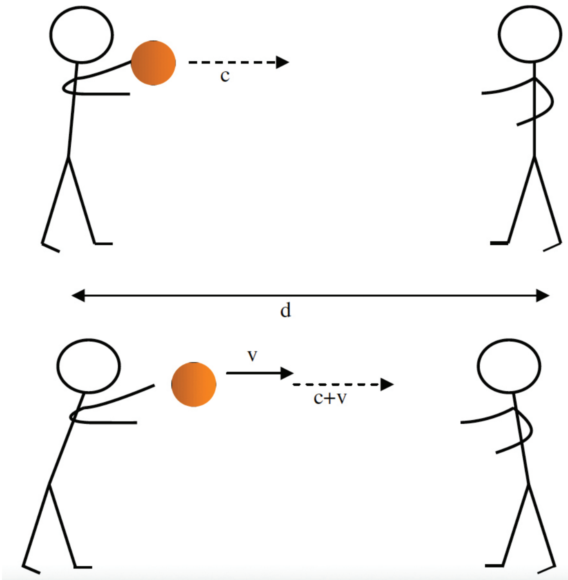

Figure 3. A boy takes and throws a ball to another one at a distance d. The second sees it before launch at a time t=d/c since light is reflected by the ball and transmitted at velocity c. When the ball is thrown with velocity v, according to the classical velocity composition or absolute time, the time would be t=d/(v+c) smaller then the previous one. Therefore the effect would precede the cause. For common velocities this is insignificant but raises a matter of principle.

experiment can determine absolute motion also by means of electromagnetic phenomena, then the wavefront of a spherical pulse of light (electromagnetic wave) emitted at the origin of S must have the same shape also in S' (from [17]).

$$x^2 + y^2 + z^2 = c^2t^2 \rightarrow x'^2 + y'^2 + z'^2 = c^2t'^2 \quad (2)$$

We denote the "fixed" system by S(x,y,z,t) and the moving one with velocity +*v* along the x axis by S'(x',y',z',t') (see Fig.4). Although completely equivalent the first one will be identified in the following mostly with the terrestrial one.

It is easy to see that this condition is not satisfied by Galilean transformations which are valid only for classical mechanics. The asymmetry due to absolute time

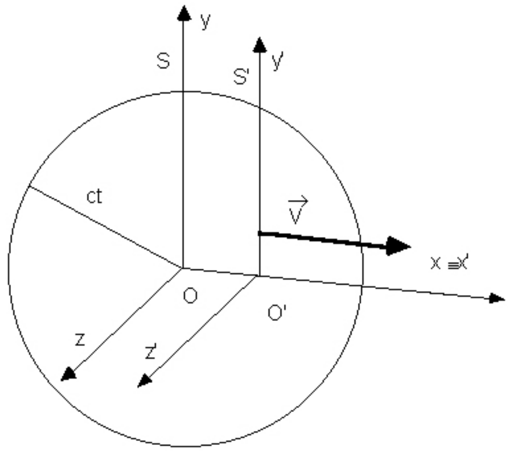

Figure 4. The non existence of absolute motion implies that light signals must have the same shape in all inertial frames.

$$\begin{cases} t' = t \\ x' = x - vt \end{cases} \tag{3}$$

is hence questioned[3].

On the contrary Eq. (1.2) is satisfied by the Lorentz transformations

$$\begin{cases} \Delta t' = \gamma(\Delta t - v/c^2 \Delta x) \\ \Delta x' = \gamma(\Delta x - v\Delta t) \end{cases} \tag{4}$$

$$\Delta y' = \Delta y \, , \; \Delta z' = \Delta z \tag{5}$$

where

$$c = c'$$

and $\gamma = 1/\sqrt{1 - v^2/c^2}$

Since the origin is arbitrary the previous relation is written in general as

$$\Delta x^2 + \Delta y^2 + \Delta z^2 = c^2 \Delta t^2$$

where $\Delta x = x_2 - x_1$ and analogously for the other coordinates.

3. To avoid confusion it must be stressed that we demand invariance of the equation and not of the solutions.

The physical content of this principle of relativity is that space and time are now intertwined and that the physically relevant quantities are determined by the invariant interval

$$\Delta s^2 = c^2\Delta t^2 - \Delta r^2$$

which generalizes the NR invariant length of a vector

$$\Delta r^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$

For differential intervals we have

$$ds_\mu = (cdt, dr)$$

and

$$ds^2 = c^2dt^2 - dr^2 = cdt'^2 - dr'^2$$

Thus *an event is specified by the 4 space-time coordinates*. In order to give unambiguous predictions for time and space intervals, one has to specify how the measurement is performed. The well known results are:

a) *time contraction in a moving frame*

Thus proper time τ results from measurements at the same space point ($\Delta x' = 0$, we consider now motion along an axis)

$$ds^2 = c^2dt^2 - dx^2 = c^2d\tau^2$$

and since $\Delta x/\Delta t$ is the velocity - v with which the other frame is moving with respect to S', τ defines *the proper time*, since no space coordinates are involved in it. It is the shortest time interval

$$t' = \tau = t\sqrt{1 - v^2/c^2} \qquad (6)$$

Well known historical example of that is the flying muon from the ionosphere: in the earth system S which propagates with velocity $-v$ with respect to the rest system of the flying muon, the time t is longer than the the proper time τ of the muon at rest. The muon can thus cover a greater distance than predicted classically.

b) *length contraction in a moving frame*

In the muon frame the earth is approaching with the said velocity and since the muon proper time is shorter, *because of the constancy of c in every frame*, the atmosphere thickness is correspondingly shorter or:

$$\Delta r' = c\Delta\tau = \Delta r\sqrt{1 - v^2/c^2} \tag{7}$$

We can thus summarize our results by saying that *in the rest frame time intervals and lengths are shorter.* In conclusion we see that the traditional (and anthropomorphic) way of looking at space and time as two separate entities has been replaced by the concept of the relativistic invariant. These results, as seen, come simply from the concept of invariant interval. Other ones, like velocity composition, require the explicit use of the Lorentz transformations. Thus we can roughy summarize the content of SR by saying that it complies with causality in inertial frames (i.e. non accelerated). Let us repeat, as it will be discussed later, that these are experimental facts, not dependent on the coordinates as in GR.

What its relevance to gravitation? The problems are summarized in Fig. 5.

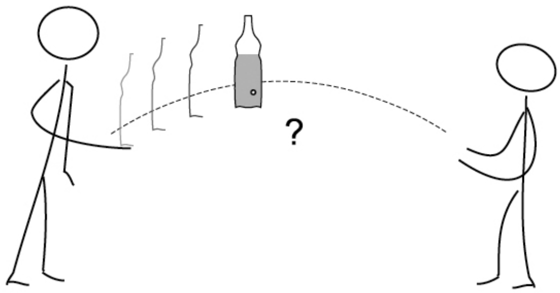

Figure 5. What does it happen to the water in the punched bottle?

Now the two boys throw each other a bottle under the influence of gravity (which was neglected in the previous example). As everybody can easily convince himself water will not flow out of the bottle. This observation[4] (in reality the equivalent considerations in a free

4. The example taken from E. Fabri [18] handouts on gravitation is more pregnant for our purposes.

falling elevator) led Einstein to assume the *principle of equivalence*. In a restricted region (locally, at the bottle) *in all free falling frames gravity can be eliminated by a proper acceleration and thus gravity and inertia are the same phenomenon*. In fact we see that water is at rest in the accelerated flying bottle[5].

But the problem remains of treating gravity at large where this cancellation cannot be achieved. In GR the SR invariant interval, which had to be introduced to account for causality, has therefore to be modified since now the flow of time depends on the height $t = t(z)$. This can be easily understood by considering a photon emitted by an atom at the height z in the famous falling elevator. At a given instant all points have the same velocity but the photon emitted by the apparatus at the top takes the time to reach the bottom for small velocities $t \simeq h/c$.

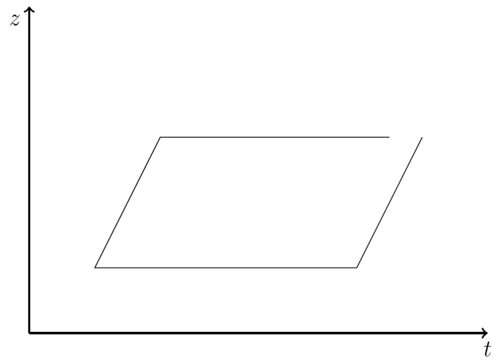

Figure 6. Time runs QUICKER at decreasing vertical coordinate z (smaller gravitational potential t = t(z). Oblique lines represent light propagation.

In the meantime however the receiver at the bottom has acquired a velocity $v = at \simeq gh/c$ so the frequency at the detector by NR Doppler effect is simply

$$\nu_{receiver,down} \simeq \nu_{emitter,up}(1 + gh/c^2)$$

5. This example is also useful to illustrate the problem of the inertial and gravitational mass. As a matter of fact it is obvious that the mass which determines the motion is the usual one i.e. the inertial one. On the other hand its gravitational properties are of course governed by the gravitational mass. Is it reasonable to accept their identity or should one conceive of a difference?

This might look like an ordinary Doppler effect but Einstein's conclusions were revolutionary. The received frequency is higher (violet shift) but this implies, since the frequency is the opposite of the flow of time that time runs differently according to the gravitational potential.

From the equivalence principle the same thing should occur for two clocks separated by the height h in a gravitational field. This is reflected in the accompanying Fig. 6). It is usually expressed intuitively by saying that of two twins living at different places, the one living in the mountains gets old before of that living at the seaside[6].

An additional argument which dispenses with the EP is the following: the emitted frequency be ω so that its energy

$$E = \hbar\omega$$

The photon of mass E/c^2gains in falling an extra gravitational energy E/c^2gh so that its energy at the bottom is

$$E = \hbar\omega(1 + gh/c^2)$$

from which the previous result.

Its extrapolation would give

$$\hbar\omega(1 - GM/c^2r) = \hbar\omega'(1 - GM/c^2(r + h))$$

i.e. the photon would reach with zero energy the point $GM/c^2R_{b.h} = 1$ (b.h. standing for black hole). According to GR (see after) on the contrary the factor in parenthesis should be $\sqrt{1 - 2GM/c^2r}$ from which the previous condition would be reached at the value $2GM/c^2R_S = 1$ double of the preceding result. The previous relations practically coincide and have been tested experimentally only at the earth surface and for GPS, but disagree completely between R_S and $R_{b.h}$. *One can thus summarize the situation as follows: in GR the Schwarzschild radius is*

6. Note the smallness of the effect. For the solar system it is of the order of 10^{-6}. *Thus the effect of curvature is indeed a small one* and the popular figures of a deformed membrane are totally misleading since no drawing could reproduce faithfully this effect. Even at cosmological scales $GM_U/c^2R_U \simeq 1$ and this might imply corrections of that order. However no deformation actually happens there, fact which is simply reproduced in this approach (see chapter 4) whereas only in a contradictory involved way in GR. The conclusion seems to be that deformation is a local effect.