ADVANCES IN TRANSPORTATION STUDIES
An International Journal

Editor in Chief: Alessandro Calvi

Vol. LXX November 2026

Contents

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Advances in Transportation Studies: an international Journal 70 (2026)

ISBN 979-12-218-2968-6 – ISSN 1824-5463 – DOI 10.53136/97912218296861 – pag. 3-20

Research on route optimization model and hybrid algorithm for one-way circular microcirculation bus routes

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subm. 8th February 2026 approv. after rev. 4th April 2026

Abstract

To address the problems of difficult coordination between route mileage and service scale and low solution efficiency in the planning of urban one-way circular microcirculation bus routes, this paper constructs an objective function model of minimizing comprehensive mileage with penalty values, designs a hybrid solution strategy integrating ant colony and Dijkstra algorithms, and verifies the effectiveness of the model and algorithm through numerical examples. The model introduces a penalty value for the mileage of a single route, abandons the constraints of minimum trip distance and minimum number of stops visited, and retains only the maximum trip constraint, thereby achieving dynamic coordination between route mileage and service scale and improving the model's scenario adaptability. On the solution side, a hybrid ant colony-Dijkstra algorithm with dynamic origin-destination selection is adopted, combined with a one-step-updated dynamic adjacency matrix and an automatic elimination strategy for bus stops along the route, significantly improving search efficiency. During the route construction process, a look-ahead judgment of the trip constraint is introduced, accurately calculating the accumulated mileage and remaining return distance based on the real-time updated adjacency matrix, ensuring that each route satisfies the maximum trip limit and eliminating infeasible trip solutions at the source. The numerical example results show that the model and algorithm can generate optimal solutions that satisfy the constraints, achieving 100% coverage of bus stops, stable convergence of the objective function, and excellent solution efficiency and practical applicability.

Keywords – one-way circular microcirculation bus, route optimization model, mileage penalty, Ant Colony-Dijkstra algorithm, dynamic adjacency matrix

1. Introduction

With the rapid advancement of urbanization and the continuous expansion of urban spatial layout, coupled with the increasing complexity of traffic scenarios [1], the contradiction between the construction of urban trunk public transport systems and residents' demand for short-distance travel has become increasingly prominent. The "last mile" travel dilemma has gradually become a key bottleneck restricting the improvement of the overall service level of urban public transport and affecting the resilience of transport networks [2]. Subways, conventional buses and other trunk public transport systems have formed relatively comprehensive coverage in urban central areas, but limited by route layouts and stop settings, there are obvious service blind spots in peripheral communities, industrial parks and areas around transport hubs, where scattered short-distance travel demands cannot be effectively met, leading to insufficient attractiveness of public transport, increased use of private cars, and further aggravation of urban traffic congestion and environmental pollution.

As an important supplement and supporting mode to urban trunk public transport, microcirculation bus specifically serves to connect trunk public transport stops with scattered travel demand points in small areas, featuring short routes, flexible operation and high coverage density, effectively filling the service gaps of trunk public transport, optimizing the spatial layout of urban bus networks, and improving the overall transfer efficiency of the bus system. In recent years, microcirculation bus, as an important measure to achieve integrated urban public transport development and green travel, has been widely promoted and applied in various cities. However, in the actual planning and operation process, microcirculation bus still faces many practical problems, such as insufficiently flexible route design, poor matching with actual travel demand, and low efficiency of route optimization algorithms. Especially for one-way circular microcirculation bus routes, which are widely used in urban communities and areas around transport hubs and favored for their high operational efficiency and strong adaptability to road networks, traditional planning methods have obvious limitations and are difficult to adapt to the dynamic and scattered characteristics of short-distance travel demand. Therefore, conducting in-depth research on optimization models and solution algorithms for one-way circular microcirculation bus routes, breaking through the rigid constraints of traditional planning models, and improving the scientificity and efficiency of route planning have important practical application value.

In the field of microcirculation bus planning, scholars have conducted in-depth research on optimization model construction under different scenarios and corresponding solution algorithm design for urban short-distance travel demand, yielding rich research results. Du et al. [3] constructed a microcirculation bus planning model based on spatiotemporal analysis of multi-modal boarding and alighting demands, and proposed an iterative greedy algorithm to reduce network transport load for route optimization. Chen et al. [4] established a bi-level mixed-integer programming model for microcirculation bus route design and frequency coordination considering shared bicycles, where the upper level minimizes the weighted sum of passenger travel cost and enterprise operating cost, and the lower level maximizes the number of passengers served, and developed a Monte Carlo adaptive genetic algorithm to improve solution efficiency and accuracy. Hu et al. [5] proposed a two-stage algorithm framework for optimizing micro-mobility service areas to maximize service demand, redefined the shareable graph for static spatial partitioning, and solved the weighted maximum coverage problem through integer programming, achieving efficient area division. Jiang et al. [6] proposed a multidimensional decision-making model for identifying microcirculation bus route areas based on the TOD concept, integrating multiple indicators such as short-term demand intensity, combined with entropy-weighted TOPSIS and clustering to divide the planning area into different traffic zones to determine priority deployment areas. Li et al. [7] used bus IC card, GPS and taxi GPS big data to construct a bi-level programming model for regional microcirculation bus route optimization, identified low-accessibility areas and candidate stops through data mining, and used genetic algorithms for route planning. Cao et al. [8] constructed constraints based on route integrity and capacity, designed a genetic algorithm-based solution method, and verified its efficiency by comparison with an enumeration algorithm. Xiong et al. [9] constructed a total cost minimization model for community shuttle bus route optimization considering user cost and supply cost, optimized routes and headways under demand and budget constraints, and compared depth-first search with genetic algorithm, proving that the latter has higher reliability and efficiency. Zhang et al. [10] constructed a two-stage optimization planning model for rural customized bus routes aiming to minimize operator and passenger costs, designed a hybrid algorithm of bat algorithm and adaptive particle swarm optimization, and verified its superiority through a real rural road network case. Yanık et al. [11] constructed a multi-objective bus route optimization model including short-turn patterns to minimize overcapacity, undercapacity and passenger time cost, verified the significant benefits of short-turn strategies through real cases, providing new ideas for flexible route design. Zhu et al. [12] established a potential demand model for feeder buses to urban rail transit, constructed a circular line model with rail stations as origins and destinations, and used genetic algorithms for route design, verifying feasibility through the Weifang community case in Shanghai. Guo et al. [13] constructed a bi-objective optimization model for high-speed rail feeder bus routes, with objectives of maximizing service demand and minimizing route length, used the ε-constraint method to generate exact Pareto optimal solutions, and analyzed the impact of maximum walking distance on the solutions. Hu et al. [14] proposed a bi-level mixed-integer programming model for the coordinated design of feeder buses and shared bicycles, developed an improved genetic algorithm embedded with fixed-point iteration for large networks, and compared joint planning with separate planning. Gao et al. [15] constructed a three-dimensional multi-objective feeder bus network optimization model considering elastic demand, proposed an improved NSGA-II algorithm with innovative encoding and adaptive parameters, and verified its good convergence and solution distribution performance through simulation. Wang et al. [16] constructed a dynamic optimization model for rural microcirculation bus networks based on a bi-level programming framework, where the upper level uses the Floyd algorithm to optimize node topology and the lower level uses an improved Logit model for stochastic user equilibrium assignment. Su et al. [17] constructed a bi-level multi-objective model for shared microcirculation bus route and frequency planning, where the upper level maximizes passenger flow and minimizes operating cost, using a customized NSGA-II to generate Pareto optimal solutions, and verified the model effectiveness through a case study. Fang et al. [18] analyzed metro feeder travel demand based on IC card big data, determined candidate stops through K-means clustering, constructed an optimization model minimizing the sum of resident cost and operating cost under multiple constraints, and used a genetic algorithm to solve the optimal circular route. Chen et al. [19] proposed a microcirculation transport network design model for arterial traffic diversion, aiming to minimize reconstruction cost and land occupation, transformed the mixed network design problem into a discrete problem through discretization, and used genetic algorithms for layout optimization.

Although the above studies have formed a relatively comprehensive theory and technology system for microcirculation bus planning from aspects such as multi-scenario model construction and multi-type algorithm design, laying a solid foundation for the development and application of microcirculation bus, there are still obvious deficiencies in specialized research on one-way circular microcirculation bus routes. Scholars have conducted extensive research on rigid constraint optimization models for microcirculation bus, and have made multiple improvements and innovations in genetic algorithms and their hybrid algorithms for solving the models, but there are few studies on flexible constraint optimization models with mileage penalty values for one-way circular routes. In terms of algorithm design, existing research mainly focuses on the improvement and application of traditional intelligent algorithms, and studies on dynamic updating of adjacency matrices and dynamic selection of origins in the optimization of one-way circular microcirculation bus routes are relatively rare.

In view of the deficiencies in existing research on one-way circular microcirculation bus routes, this paper takes one-way circular microcirculation bus as the research object, conducts in-depth research on the construction of its optimization model and the design of solution algorithms, focusing on solving the rigid constraint problem of traditional models and the low efficiency of algorithm solution, so as to improve the scientificity and flexibility of one-way circular microcirculation bus route planning.

2. Model construction

2.1. Assumptions

The microcirculation bus routes studied in this paper adopt a one-way circular operation mode. The following assumptions are made in the modeling and optimization process:

(1) In the modeling, only the spatial locations of stops are considered, without pre-distinguishing whether a stop is located on the left or right side of the road. After the route direction is determined, all stops are uniformly placed on the right side of the vehicle travel direction to comply with the actual operation norm of right-hand driving.

(2) The route distance is calculated solely based on the spatial coordinates of stops and the road network topology, without considering minor mileage deviations caused by the left-right positional differences of stops.

(3) To align with the actual operation habit of right-hand driving for one-way circular microcirculation buses in Chinese cities, this paper uniformly adopts the clockwise direction as the final route travel direction. If the stop sequence output by the optimization algorithm is counterclockwise, only a reverse order reversal of the sequence is needed to obtain a clockwise route scheme that conforms to actual operation rules, without affecting the rationality of the model or the validity of the results.

(4) The connectivity of the road network within the study area is fixed, and the direct connectivity relationships between nodes as well as node coordinates are known constants. The impacts of dynamic traffic conditions such as road construction and traffic congestion on travel are not considered in the modeling and solution process.

(5) All bus stops have basic stopping conditions and can meet the stopping and passenger flow distribution demands of microcirculation buses, with no restrictions on stop capacity or stopping duration.

(6) Microcirculation bus routes only travel between the planned road network nodes within the study area, do not cross the study area boundary, and vehicles only pick up and drop off passengers at bus stops. Traffic nodes serve only as transit passage nodes and do not function as passenger distribution points.

2.2. Variable and parameter definition

The set of candidate origin-destination bus stops for one-way circular microcirculation bus routes is $U_1 = \{1,2,\dots,m_1\}$. When constructing a route, each circular route selects only one stop from U_1 as its sole origin-destination stop; the remaining stops in this set that are not selected as the origin-destination stop can still be included in the route, but only as ordinary passing stops without functioning as origin-destination stops. The set of rail transit feeder bus stops is $U_2 = \{m_1 + 1, m_1 + 2, \dots, m_2\}$, the set of arterial road feeder bus stops is $U_3 = \{m_2 + 1, m_2 + 2, \dots, m_3\}$, and the set of general bus stops is $U_4 = \{m_3 + 1, m_3 + 2, \dots, m_4\}$. The set of all bus stops is $U = U_1 \cup U_2 \cup U_3 \cup U_4$. The set of traffic nodes is $V = \{m_4 + 1, m_4 + 2, \dots, n\}$. Let W represent the set of road network nodes, which includes both bus stops and traffic nodes, i.e., $W = U \cup V$.

Let i, j be node indices. Let k be the index of a bus route, and the set of bus routes is $K = \{1,2,\dots,K_{\max}\}$, where $K_{\max}$ denotes the maximum allowable number of bus routes. This parameter is only used to limit the scale of routes; in the actual execution of the algorithm, the number of routes is dynamically generated through the iterative process (until all bus stops are traversed).

$x_{i,j}^k$ is a 0-1 variable indicating whether the k-th route selects the segment from node i to node j. If yes, the value is 1; otherwise, 0. y^k is a 0-1 variable indicating whether the k-th route is operated. If yes, the value is 1; otherwise, 0. y_i^k is a 0-1 variable specifying whether bus route k passes through node i; if yes, the value is 1; otherwise, 0.

$y_{i,j}$ is a road network connectivity parameter. If a directly connected segment exists between node i and node j, then $y_{i,j} = 1$; otherwise, $y_{i,j} = 0$. $l_{i,j}$ denotes the path distance from node i to node j.

M is a large constant. γ_1 and γ_2 are given positive integers satisfying $1 \leq \gamma_1 \leq |U_2|$ and $1 \leq \gamma_2 \leq |U_3|$.

λ is the mileage penalty value for a single route, with physical unit in meters. For each newly added operating route, a fixed mileage penalty value λ is added to the objective function, thereby suppressing excessive growth in the number of routes and achieving collaborative optimization between route mileage and route scale.

H_k denotes the set of all nodes traversed by route k.

$L_{\max}$ is the maximum allowable travel mileage for a route.

2.3. Objective function and constraint construction

The comprehensive mileage minimization objective function in this paper can be expressed as Equation (1). The constraints are given in Equations (2)-(14).

$$\min Z = \sum_{k\in K}\sum_{i\in W}\sum_{j\in W} x_{i,j}^k \cdot l_{i,j} + \lambda \sum_{k\in K} y^k \tag{1}$$

$$y_i^k \leq y^k \quad \forall k, \forall i \in W \tag{2}$$

$$x_{i,j}^k \leq y_i^k \quad \forall k, \forall i, j \in W \tag{3}$$

$$x_{i,j}^k \leq y_{i,j} \quad \forall k, \forall i, j \in W \tag{4}$$

$$\sum_{k\in K} y_i^k \geq 1 \quad \forall i \in U \tag{5}$$

$$1 - M(1 - y^k) \leq \sum_{i\in U_1} y_i^k \quad \forall k \tag{6}$$

$$\gamma_1 - M(1 - y^k) \leq \sum_{i\in U_2} y_i^k \quad \forall k \tag{7}$$

$$\gamma_2 - M(1 - y^k) \leq \sum_{i\in U3} y_i^k \quad \forall k \tag{8}$$

$$\sum_{i\in W}\sum_{j\in W} x_{i,j}^k \cdot l_{i,j} \leq L_{\max} \quad \forall k \tag{9}$$

$$\sum_{i\in W} x_{i,b}^k = y_b^k \quad \forall k, \forall b \in W \tag{10}$$

$$\sum_{j \in W} x_{b,j}^{k} = y_{b}^{k} \quad \forall k, \forall b \in W \tag{11}$$

$$H_k = \{ i \in W \mid y_i^k = 1 \} \quad \forall k \tag{12}$$

$$\sum_{i \in D} \sum_{j \in D} x_{i,j}^{k} \leq |D| - 1 \quad \forall D \text{ Ö } H_k, D \neq \varnothing, \forall k \tag{13}$$

$$x_{i,j}^{k}, y_{i}^{k}, y^{k} \in \{0,1\} \tag{14}$$

Equations (2)-(3) are route-node association constraints. Equation (2) indicates that only when route k is operated ($y^k = 1$) can node i be assigned to that route. Equation (3) indicates that only when node i is assigned to route k ($y_i^k = 1$) can route k pass through the segment from i to j ($x_{i,j}^k = 1$). Equation (4) is the road network connectivity constraint. Its core function is to restrict bus routes to travel only on existing road segments. Only when a directly connected physical segment exists between node i and node j (i.e., $y_{i,j} = 1$) can route k select the segment from i to j (i.e., $x_{i,j}^k$ can take the value 1). Equation (5) is the bus stop coverage constraint. All bus stops in set U are visited at least once by an operated bus route, ensuring full coverage of bus service. Equations (6)-(8) are route stop composition constraints, where M is a large constant; when $y^k = 0$ the constraints are automatically relaxed; when $y^k = 1$ the constraints take effect. Equation (6): if route k is operated ($y^k = 1$), it must include at least one stop from the candidate origin-destination stop set U_1. Equation (7): if route k is operated, it must include at least γ_1 stops from the rail transit feeder stop set U_2. Equation (8): if route k is operated, it must include at least γ_2 stops from the arterial road feeder stop set U_3. Equation (9) is the maximum travel mileage constraint for a single route. The total travel mileage of a single bus route shall not exceed the prescribed upper limit $L_{\max}$, ensuring route operational efficiency and feasibility. Equations (10)-(11) are flow conservation constraints. For any node b traversed by route k ($y_b^k = 1$), the number of incoming segments equals the number of outgoing segments, both equal to 1, ensuring the circular structure and path continuity of the route; if node b is not traversed by route k ($y_b^k = 0$), both incoming and outgoing numbers are 0. Equation (12) defines the set of route nodes. It defines H_k as the set of all nodes traversed by route k, i.e., the set of all nodes i with $y_i^k = 1$. Equation (13) is the subtour elimination constraint. It eliminates subtours in route k, ensuring that the route has a single circular structure. For any non-empty proper subset D of H_k, the number of internal segments does not exceed $|D| - 1$, avoiding the formation of local closed loops. Equation (14) is the variable type constraint. All decision variables are 0-1 binary variables.

3. Solution method

Let t denote the iteration round, with the range $t = 1,2, \dots, T$; let s denote the ant index, with $s = 1,2, \dots, c$. Let f denote the number of steps of stops already selected by an ant during route construction, with $f = 1,2$, ..., where $f = 1$ indicates that the ant has completed the selection of the first stop (origin-destination stop). Unless otherwise specified, the ranges of t, s, f in this chapter follow the above conventions and will not be further explained. Also define $K_{t,s}^*$ as the number of routes actually constructed by the s-th ant in the t-th iteration. In this chapter, the range of route index k is $k = 1,2, \ldots, K_{t,s}^{*}$. Unless otherwise specified, the range of k in this chapter follows the above convention. Define the set $U_{t,s}^{k,f}$, which denotes the set of remaining target bus stops available for the next step when the s-th ant has taken f steps forward in constructing route k during the t-th iteration. In the initial state, let $U_{t,s}^{1,0} = U$, indicating that in the t-th iteration, ant s is at the initial state of constructing the first route, where all bus stops are selectable. Define $U_{t,s}^{k,0}$ as the set of all bus stops that remain unvisited and can be used to construct the k-th route after the s-th ant completes the construction of the $(k-1)$-th route in the t-th iteration.

3.1. Selection method for origin-destination stops

To quantify the spatial aggregation advantage of candidate origin-destination stops, for the k-th route constructed by any ant s in any iteration round t, the heuristic function for selecting node i as the origin-destination stop is defined as $\eta_i(t,s,k)$, which measures the spatial suitability of candidate origin-destination point i as the origin-destination stop for that route. Its calculation formulas are shown in Equations (15)-(17):

$$\eta_i(t,s,k) = \frac{1}{\bar{d}_i(t,s,k)} \quad i \in U_1 \tag{15}$$

$$\bar{d}_i(t,s,k) = \frac{1}{|U_{t,s}^{k,0} \setminus \{i\}|} \sum_{j \in U_{t,s}^{k,0} \setminus \{i\}} d_{i,j} \quad i \in U_1 \tag{16}$$

$$d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad i,j \in U \tag{17}$$

Equation (15) is the heuristic function for selecting the origin-destination stop of a route. Here, $\bar{d}_i(t,s,k)$ represents, when ant s selects the origin-destination stop for route k in the t-th iteration, the average Euclidean distance from candidate origin-destination point i to all currently unvisited bus nodes. If the candidate starting point i itself is included in the set of unvisited bus stops, it is excluded from the distance calculation.

Equation (16) is the calculation formula for $\bar{d}_i(t,s,k)$. $d_{i,j}$ denotes the Euclidean distance between node i and node j.

Equation (17) is the calculation formula for $d_{i,j}$. In this formula, x_i, y_i are respectively the abscissa and ordinate of bus node i; x_j, y_j are respectively the abscissa and ordinate of bus node j.

Define $P_i(t,s,k)$ as the probability that bus stop i is selected as the origin-destination stop, meaning: in the t-th iteration, ant s selects bus stop i as the origin-destination stop for route k. The calculation formula is shown in Equation (18). In actual solution, a selection interval is constructed according to this probability distribution, and random sampling is performed using the roulette wheel strategy, thereby determining the origin-destination stop of route k in this iteration.

$$P_i(t,s,k) = \frac{\eta_i(t,s,k)}{\sum_{a \in U_1} \eta_a(t,s,k)} \quad i \in U_1 \tag{18}$$

Define $v_{\text{dep}}^{t,s,k}$ as the selected origin-destination stop when the s-th ant constructs route k in the t-th iteration. Define $\xi_{t,s}^{k,f}$ as a sequence recording all nodes (including bus stops and traffic nodes) passed by ant s after taking f steps forward while constructing route k in the t-th iteration. After the origin-destination stop is determined, i.e., the ant advances one step, $\xi_{t,s}^{k,1} = (v_{\text{dep}}^{t,s,k})$, and the set of bus stops to be visited $U_{t,s}^{k,1}$ is updated as: $U_{t,s}^{k,1} = U_{t,s}^{k,0} \backslash \{v_{\text{dep}}^{t,s,k}\}$.

3.2. Selection method for target bus stops

Define $\eta_{i,j}$ as the heuristic function from the current stop i to the candidate target bus stop j, with its expression shown in Equation (19).

$$\eta_{i,j} = \frac{1}{d_{i,j}} \quad j \in U \tag{19}$$

Define $P_{i,j}(t,s,k,f)$ as the state transition probability that, in the t-th iteration, when ant s has taken f steps forward in constructing route k and is currently at bus stop i, it selects the next target stop j to visit from the current stop i. The calculation formula for $P_{i,j}(t,s,k,f)$ is shown in Equation (20). In actual solution, instead of directly selecting the stop with the highest probability, random sampling is performed using the roulette wheel method based on the state transition probability distribution to determine the next bus stop j to visit.

$$P_{i,j}(t,s,k,f) = \begin{cases} \dfrac{[\tau_{i,j}(t)]^{\alpha} \cdot [\eta_{i,j}]^{\beta}}{\sum\limits_{h \in U_{t,s}^{k,f}} [\tau_{i,h}(t)]^{\alpha} \cdot [\eta_{i,h}]^{\beta}} & j \in U_{t,s}^{k,f} \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

In Equation (20), $\tau_{i,j}(t)$ represents the pheromone concentration on edge (i,j) in the t-th iteration. α is the pheromone importance factor, controlling the influence weight of historical path experience ($\alpha > 0$). β is the heuristic function importance factor, controlling the influence weight of spatial proximity ($\beta > 0$).

The pheromone concentration update formulas for edge (i,j) are shown in Equations (21)-(23).

$$\tau_{i,j}(t+1) = (1-\rho)\tau_{i,j}(t) + \sum_{s=1}^{c} \Delta\tau_{i,j}^{t,s}, \quad 0 < \rho < 1 \quad i,j \in U \tag{21}$$

$$\Delta\tau_{i,j}^{t,s} = \sum_{k=1}^{K_{t,s}^{*}} \Delta\tau_{i,j}^{t,s,k} \quad i,j \in U \tag{22}$$

$$\Delta\tau_{i,j}^{t,s,k} = \begin{cases} \dfrac{Q}{\sum\limits_{k'=1}^{K_{t,s}^{*}} L_{\text{total}}(t,s,k') + \lambda \cdot K_{t,s}^{*}} & \text{Route } k \text{ passes through edge } (i,j). \\ 0 & \text{otherwise} \end{cases} \quad i,j \in U \tag{23}$$

In Equation (21), $\Delta\tau_{i,j}^{t,s}$ represents the total pheromone increment released by ant s on edge (i,j) in the t-th iteration, which is calculated by Equation (22). ρ is the pheromone evaporation factor ($0 < \rho < 1$), representing the proportion of pheromone that evaporates in each iteration. In Equation (22), $\Delta\tau_{i,j}^{t,s,k}$ represents the pheromone increment released by ant s on edge (i,j) when constructing route k in the t-th iteration, which is calculated by Equation (23). In Equation (23), $L_{\text{total}}(t,s,k')$ denotes the total length of the k'-th bus route constructed by the s-th ant in the t-th iteration; Q is the preset total pheromone constant, used to control the total scale of pheromone released in a single iteration.

3.3. Path selection between bus stops and actual distance calculation

After the origin-destination stop of a route is determined in Section 3.1 and the target bus stop to visit is determined in Section 3.2, since bus stops are often not directly connected and require transit through several nodes to achieve connectivity, it is necessary to construct a path selection scheme between bus stops and calculate the corresponding actual travel distance.

In this paper, the Dijkstra algorithm is adopted to solve the path selection problem between bus stops and to determine the path. The specific algorithm description is as follows.

Define $v_{\text{cur}}^{t,s,k,f}$ as the current bus stop where ant s is located after taking f steps forward while constructing route k in the t-th iteration. When $f = 1$, the current bus stop is the origin-destination stop, i.e., $v_{\text{cur}}^{t,s,k,1} = v_{\text{dep}}^{t,s,k}$. $v_{\text{tar}}^{t,s,k,f}$ is the preselected next target stop from the current bus stop $v_{\text{cur}}^{t,s,k,f}$ by ant s.

Define the dynamic adjacency matrix $A_{t,s}^{k,f} = [a_{i,j}(t,s,k,f)]$, where $A_{t,s}^{k,f}$ represents the dynamic adjacency matrix after ant s has taken f steps in constructing the k-th route in the t-th iteration, and $a_{i,j}(t,s,k,f)$ is an element of this matrix, indicating the travel distance from node i to node j in the current state, with $i,j \in W$. When $f = 1$, it indicates that ant s is at the starting point, corresponding to the initial state of path search, and the elements of the dynamic adjacency matrix $a_{i,j}(t,s,k,1)$ are reset to the static initial values $a_{i,j}$, i.e., $a_{i,j}(t,s,k,1) = a_{i,j}$. This serves as the initial condition for constructing this route. Here, $a_{i,j}$ is the initial travel distance from node i to node j, and the value of $a_{i,j}$ is given by Equation (24). The meaning of this equation is: if node i and node j are directly connected, then take the actual segment length $l_{i,j}$; otherwise, take ∞, indicating impassability.

$$a_{i,j} = \begin{cases} l_{i,j} & y_{i,j} = 1 \\ \infty & y_{i,j} = 0 \end{cases} \quad i,j \in W \tag{24}$$

Define the shortest path function as $\text{Dijkstra}(v_{\text{cur}}^{t,s,k,f}, v_{\text{tar}}^{t,s,k,f}, A_{t,s}^{k,f})$. The mathematical meaning of this function is: taking $v_{\text{cur}}^{t,s,k,f}$ as the starting point and $v_{\text{tar}}^{t,s,k,f}$ as the ending point, based on the adjacency matrix $A_{t,s}^{k,f}$, through a greedy strategy that sequentially selects the unvisited node closest to the starting point and updates the shortest path estimation value for each node, finally obtaining the unique shortest distance $D(v_{\text{cur}}^{t,s,k,f}, v_{\text{tar}}^{t,s,k,f}, A_{t,s}^{k,f})$ and the shortest path sequence $W(v_{\text{cur}}^{t,s,k,f}, v_{\text{tar}}^{t,s,k,f}, A_{t,s}^{k,f})$. Its calculation process can be expressed as Equation (25).

$$(D(v_{\text{cur}}^{t,s,k,f}, v_{\text{tar}}^{t,s,k,f}, A_{t,s}^{k,f}), W(v_{\text{cur}}^{t,s,k,f}, v_{\text{tar}}^{t,s,k,f}, A_{t,s}^{k,f})) = \text{Dijkstra}(v_{\text{cur}}^{t,s,k,f}, v_{\text{tar}}^{t,s,k,f}, A_{t,s}^{k,f}) \tag{25}$$

The sequence $W(v_{\text{cur}}^{t,s,k,f}, v_{\text{tar}}^{t,s,k,f}, A_{t,s}^{k,f})$ contains all nodes (including bus nodes and traffic nodes) passed from $v_{\text{cur}}^{t,s,k,f}$ to $v_{\text{tar}}^{t,s,k,f}$. The first element of this sequence is $v_{\text{cur}}^{t,s,k,f}$, and the last element is $v_{\text{tar}}^{t,s,k,f}$. Meanwhile, denote $W'(v_{\text{cur}}^{t,s,k,f}, v_{\text{tar}}^{t,s,k,f}, A_{t,s}^{k,f})$ as the subsequence of $W(v_{\text{cur}}^{t,s,k,f}, v_{\text{tar}}^{t,s,k,f}, A_{t,s}^{k,f})$ after removing the first element. Denote $\overline{W}(v_{\text{cur}}^{t,s,k,f}, v_{\text{tar}}^{t,s,k,f}, A_{t,s}^{k,f})$ as the set of all elements in the sequence $W(v_{\text{cur}}^{t,s,k,f}, v_{\text{tar}}^{t,s,k,f}, A_{t,s}^{k,f})$.

3.4. Mileage constraint judgment and route construction

Define $L_{\text{acc}}(t,s,k,f)$ as the accumulated mileage of the s-th ant, the k-th route, after taking f steps forward in the t-th iteration. When constructing each route, the selected origin-destination stop is taken as the first step, and the accumulated mileage at the starting point is recorded as 0, i.e., $L_{\text{acc}}(t,s,k,1)=0$.

$v_{\text{tar}}^{t,s,k,f}$ must satisfy Equation (26) to be a feasible target stop. Equation (26) is the maximum trip constraint.

$$L_{\text{acc}}(t,s,k,f)+D\left(v_{\text{cur}}^{t,s,k,f},v_{\text{tar}}^{t,s,k,f},A_{t,s}^{k,f}\right)+D\left(v_{\text{tar}}^{t,s,k,f},v_{\text{dep}}^{t,s,k},A_{t,s}^{k,f+1}\right)\le L_{\max} \tag{26}$$

If $v_{\text{tar}}^{t,s,k,f}$ satisfies the condition of Equation (26), it is a feasible target stop and can proceed to step $f+1$, i.e.:

$$L_{\text{acc}}(t,s,k,f+1)=L_{\text{acc}}(t,s,k,f)+D\left(v_{\text{cur}}^{t,s,k,f},v_{\text{tar}}^{t,s,k,f},A_{t,s}^{k,f}\right) \tag{27}$$

$$v_{\text{cur}}^{t,s,k,f+1}=v_{\text{tar}}^{t,s,k,f} \tag{28}$$

$$a_{i,j}(t,s,k,f+1)=\begin{cases}\infty & i\in\overline{W}(v_{\text{cur}}^{t,s,k,f},v_{\text{tar}}^{t,s,k,f},A_{t,s}^{k,f})\setminus v_{\text{tar}}^{t,s,k,f}\\ a_{i,j}(t,s,k,f) & \text{otherwise}\end{cases}\quad j\in W \tag{29}$$

$$a_{i,j}(t,s,k,f+1)=\begin{cases}\infty & j\in\overline{W}(v_{\text{cur}}^{t,s,k,f},v_{\text{tar}}^{t,s,k,f},A_{t,s}^{k,f})\\ a_{i,j}(t,s,k,f) & \text{otherwise}\end{cases}\quad i\in W \tag{30}$$

$$A_{t,s}^{k,f+1}=[a_{i,j}(t,s,k,f+1)]\quad i,j\in W \tag{31}$$

$$U_{t,s}^{k,f+1}=U_{t,s}^{k,f}\setminus\overline{W}(v_{\text{cur}}^{t,s,k,f},v_{\text{tar}}^{t,s,k,f},A_{t,s}^{k,f}) \tag{32}$$

$$\xi_{t,s}^{k,f+1}=\xi_{t,s}^{k,f}\;\|\;W'(v_{\text{cur}}^{t,s,k,f},v_{\text{tar}}^{t,s,k,f},A_{t,s}^{k,f}) \tag{33}$$

Equation (27) is used to calculate the accumulated mileage of the ant at step $f+1$. Equation (28) is used to update the currently selected target node as the current node at step $f+1$. Equations (29)-(30) update the elements of the adjacency matrix corresponding to step $f+1$; their combined effect is: for nodes in the set $\overline{W}\left(v_{\text{cur}}^{t,s,k,f},v_{\text{tar}}^{t,s,k,f},A_{t,s}^{k,f}\right)$, only the target stop $v_{\text{tar}}^{t,s,k,f}$ is allowed to have one-way outward travel, no node is allowed to enter the target stop from any direction, and all other nodes in the set are prohibited from two-way travel; the remaining nodes not in the set retain their original accessibility. Equation (31) implements the overall update of the adjacency matrix for step $f+1$. Equation (32) serves to delete visited bus stops. The deletion strategy is: remove all bus stops along the entire path from the current stop to the target stop at once, rather than removing only the target stop. In this way, the scale of candidate stops at each step of the algorithm is greatly reduced, the search space is effectively decreased, and the overall solution efficiency is significantly improved. Equation (33) is the path sequence concatenation operation. In the equation, "‖" denotes the sequence concatenation operation, through which the original path is merged with the newly extended subpath, finally forming the complete path sequence of the ant after taking step $f+1$. The innovation of this concatenation operation is that it does not simply directly add the target stop to the path, but concatenates all nodes (including bus stops and traffic nodes) along the path from the current stop to the target stop as a whole into the path, thereby constructing a complete feasible path that conforms to the actual road network.

If $v_{\text{tar}}^{t,s,k,f}$ does not satisfy the condition of Equation (26), then ant s is directly made to return from the current node $v_{\text{cur}}^{t,s,k,f}$ to the origin-destination stop, forming a complete closed circular route k, and the construction of this route k is thus completed. The calculation of the total mileage of route k is shown in Equation (34), the complete path expression is shown in Equation (35), and the method for updating the initial set of unvisited bus nodes for the $(k+1)$-th route constructed by ant s in the t-th iteration is shown in Equation (36).

$$L_{\text{total}}(t,s,k) = L_{\text{acc}}(t,s,k,f) + D\left(v_{\text{cur}}^{t,s,k,f}, v_{\text{dep}}^{t,s,k}, A_{t,s}^{k,f}\right) \tag{34}$$

$$\xi_{t,s}^{k} = \xi_{t,s}^{k,f} \parallel W'(v_{\text{cur}}^{t,s,k,f}, v_{\text{dep}}^{t,s,k}, A_{t,s}^{k,f}) \tag{35}$$

$$U_{t,s}^{k+1,0} = U_{t,s}^{k,f} \setminus \bar{W}(v_{\text{cur}}^{t,s,k,f}, v_{\text{dep}}^{t,s,k}, A_{t,s}^{k,f}) \tag{36}$$

Equation (34) is used to calculate the total travel mileage of the current closed route k. In Equation (35), $\xi_{t,s}^{k}$ represents the complete path sequence of the k-th bus route constructed by ant s in the t-th iteration. The function of this equation is to complete the concatenation of the full path for the current route k, merging the sequence of traveled nodes with the path sequence returning to the origin-destination stop, thereby generating a closed circular route to record all nodes traversed by route k. Equation (36) updates the set of unvisited bus stops: it removes the bus stops covered by the path returning to the origin-destination stop from the current set of unvisited stops, obtaining the initial set of unvisited stops for constructing the next route, providing the set of available stops for subsequent ant iterations to build new routes.

When $U_{t,s}^{k,f} = \emptyset$, it indicates that the s-th ant has completed the construction of all bus routes, and the current k is the actual total number of routes $K_{t,s}^{*}$.

3.5. Judgment of number of feeder stops and number of operated routes

In the same iteration, the judgment of the number of feeder stops is performed uniformly after all ants have completed the construction of bus routes. Define the traffic node removal function as $\text{filter}(\xi_{t,s}^{k}, V)$. The mathematical meaning of this function is: remove all traffic nodes contained in set V from the original path sequence $\xi_{t,s}^{k}$, retain only bus stops while preserving the original order of bus stops in the sequence, and finally output the filtered subsequence $\underline{\xi}_{t,s}^{k}$, i.e., $\underline{\xi}_{t,s}^{k} = \text{filter}(\xi_{t,s}^{k}, V)$. Denote $\bar{\underline{\xi}}_{t,s}^{k}$ as the set of all elements in the sequence $\underline{\xi}_{t,s}^{k}$. If the number of rail transit feeder stops of route k satisfies Equation (37) and the number of arterial road feeder stops satisfies Equation (38), then route k meets the requirements; otherwise, it does not meet the requirements. If the k-th bus route constructed by the s-th ant does not meet the requirements, then all bus routes constructed by that ant are discarded. The actual number of routes $K_{t,s}^{*}$ constructed by the s-th ant in the t-th iteration must satisfy the constraint of Equation (39), i.e., the actual number of routes is not greater than the maximum allowable number of routes $K_{\max}$. Routes that do not satisfy this constraint are directly discarded.

$$|\bar{\underline{\xi}}_{t,s}^{k} \cap U_2| \geq \gamma_1 \tag{37}$$

$$|\bar{\underline{\xi}}_{t,s}^{k} \cap U_3| \geq \gamma_2 \tag{38}$$

$$K_{t,s}^{*} \leq K_{\max} \tag{39}$$

4. Case study analysis

4.1. Basic data

4.1.1. Road network structure

Figure 1 shows the road network topology of the study area, comprising a total of 80 nodes, with nodes connected by straight line segments to represent physical adjacency relationships. Based on functional attributes, nodes are divided into two categories: one category is bus stops, identified by hollow circles, numbered 1 to 29; the other category is traffic nodes, identified by solid black dots, numbered 30 to 80. Among these, bus stops are further subdivided into four types: numbers 1 to 4 are candidate origin-destination stops, 5 to 8 are rail transit feeder stops, 9 to 12 are arterial road feeder stops, and 13 to 29 are ordinary bus stops within the area. Traffic nodes are physical intersections of the road network and do not serve passenger distribution functions. In Figure 1, if a direct connecting edge exists between two nodes, they are considered adjacent and direct travel is possible (i.e., $y_{i,j} = 1$); if travel can only be achieved via transit through other nodes, they are considered non-adjacent (i.e., $y_{i,j} = 0$).

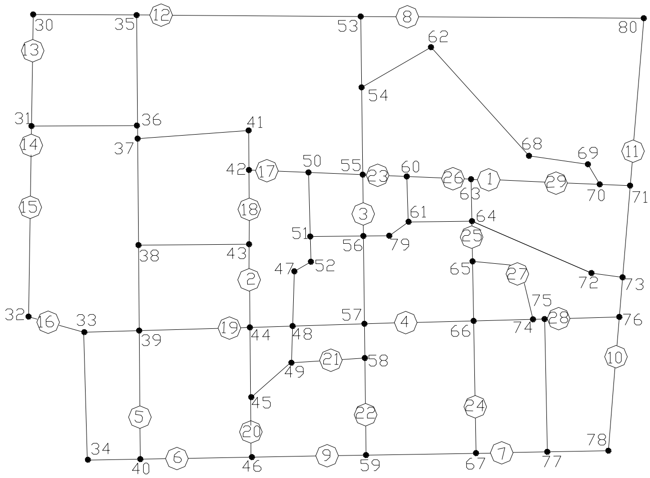

Fig. 1 - Road network diagram

4.1.2. Relevant parameter values

The relevant parameter values of the model and algorithm in this paper are shown in Table 1.

Tab. 1 - Relevant parameter values

c	α	β	ρ	Q	T	$L_{\max}$	γ_1	γ_2	λ	$K_{\max}$
60	1	5	0.2	20000	100	5500	1	1	2000	10

4.1.3. Node coordinates

The spatial coordinate parameters of each node are shown in Table 2. These coordinates are model coordinates used for model construction and are not actual geographic coordinates. When a directly connected relationship exists between two nodes (i.e., $y_{i,j} = 1$), the value of $l_{i,j}$ can be accurately calculated from their coordinates using the Euclidean distance formula, with the calculation result in meters.

Tab. 2 - Spatial coordinates of each node

No.	x_i	y_i	No.	x_i	y_i	No.	x_i	y_i
1	1164.526	715.479	28	1341.357	361.165	55	845.822	728.608
2	558.167	460.405	29	1334.907	706.961	56	847.394	572.088
3	846.901	628.375	30	11.74	1137.235	57	850.091	348.107
4	954.441	351.109	31	7.407	852.586	58	851.143	260.7
5	281.644	109.932	32	0.007	366.478	59	854.124	13.118
6	375.348	4.432	33	141.198	326.738	60	957.185	724.005
7	1197.455	18.973	34	150.135	1.227	61	961.282	607.976
8	958.528	1131.426	35	273.46	1135.625	62	1018.568	1053.772
9	755.389	11.434	36	274.25	853.858	63	1119.964	716.936
10	1486.597	264.144	37	276.003	820.512	64	1121.888	609.877
11	1529.657	788.505	38	278.195	548.818	65	1123.73	507.363
12	335.328	1135.686	39	279.955	330.718	66	1126.46	355.443
13	10.332	1044.738	40	282.861	2.756	67	1132.522	18.168
14	6.656	803.262	41	556.989	841.381	68	1266.61	776.609
15	4.252	645.345	42	557.828	740.51	69	1415.724	755.031
16	49.471	352.055	43	558.203	551.078	70	1445.591	703.821
17	603.017	738.642	44	560.236	338.835	71	1522.465	700.644
18	558.606	640.085	45	563.746	160.883	72	1424.683	477.514
19	508.246	337.267	46	565.384	7.906	73	1503.44	466.603
20	562.414	72.26	47	672.729	481.902	74	1276.704	359.396
21	765.014	255.107	48	668.271	342.291	75	1306.05	360.168
22	852.884	116.108	49	665.247	247.614	76	1495.203	365.578
23	883.296	727.059	50	708.483	734.284	77	1312.925	21.44
24	1130.404	136.283	51	712.929	570.815	78	1467.481	24.244
25	1121.283	569.173	52	714.69	506.375	79	913.022	572.709
26	1074.019	718.225	53	840.65	1132.137	80	1557.183	1127.729
27	1237.006	475.207	54	842.967	951.356			

4.2. Calculation results

Through iterative solution of the algorithm, the minimum objective function value obtained in this paper is 13689.29 meters. This optimal scheme consists of two circular microcirculation bus routes, with Route 1 having a mileage of 4375.29 meters and Route 2 having a mileage of 5314 meters, resulting in a total travel mileage of 9689.29 meters for the two routes. Meanwhile, as the penalty value for each route is set to 2000 meters, the total penalty term for the routes amounts to 4000 meters, ultimately yielding a global optimal objective function value of 13689.29 meters.

The travel path of Route 1 is: 2 → 43 → 18 → 42 → 17 → 50 → 55 → 23 → 60 → 26 → 63 → 64 → 25 → 65 → 27 → 74 → 75 → 28 → 76 → 10 → 78 → 77 → 7 → 67 → 24 → 66 → 4 → 57 → 58 → 21 → 49 → 45 → 20 → 46 → 6 → 40 → 5 → 39 → 19 → 44 → 2. This travel path includes both bus stops and traffic nodes. After removing the traffic nodes, the bus stop visit sequence of Route 1 is as follows: 2 → 18 → 17 → 23 → 26 → 25 → 27 → 28 → 10 → 7 → 24 → 4 → 21 → 20 → 6 → 5 → 19 → 2. Route 1 contains a total of 17 bus stops, including 3 rail transit feeder stops and 1 arterial road feeder stop. The alignment of Route 1 in the road network is shown in Figure 2.

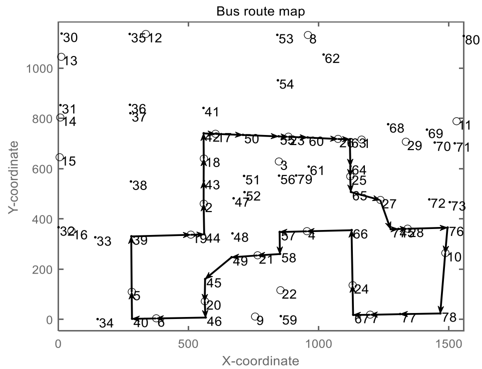

Fig. 2 - Path diagram of bus Route 1

The travel path of Route 2 is: 3 → 56 → 57 → 58 → 22 → 59 → 9 → 46 → 6 → 40 → 34 → 33 → 16 → 32 → 15 → 14 → 31 → 13 → 30 → 35 → 12 → 53 → 8 → 80 → 11 → 71 → 70 → 29 → 1 → 63 → 26 → 60 → 23 → 55 → 3. This travel path also includes both bus stops and traffic nodes. After removing the traffic nodes, the bus stop visit sequence of Route 2 is as follows: 3 → 22 → 9 → 6 → 16 → 15 → 14 → 13 → 12 → 8 → 11 → 29 → 1 → 26 → 23 → 3. Route 2 contains a total of 15 bus stops, including 2 rail transit feeder stops and 3 arterial road feeder stops. The alignment of Route 2 in the road network is shown in Figure 3.

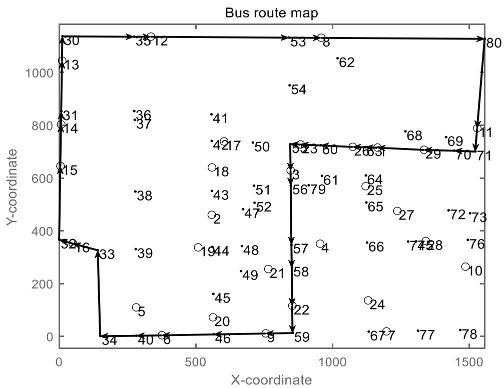

Fig. 3 - Path diagram of bus Route 2

Combining the stop coverage of Route 1 and Route 2, the two routes have achieved full coverage of all bus stops numbered 1 to 29, with an overall bus stop coverage rate of 100%. The bus stops repeatedly covered by both routes are No. 6, No. 23, and No. 26, totaling 3 stops, with a repetition rate of approximately 10.34%. For bus stops covered by both routes, the stopping strategy can be further optimized in subsequent operational scheduling based on stop spacing and passenger flow characteristics. The optimization of bus stop operations is beyond the scope of this paper and will not be discussed in depth.

The convergence curve of the algorithm calculation process is shown in Figure 4. This figure illustrates the convergence curve of the objective function value during the algorithm iteration process, with the horizontal axis representing the number of iterations and the vertical axis representing the objective function value, intuitively reflecting the optimization efficiency and convergence stability of the algorithm.

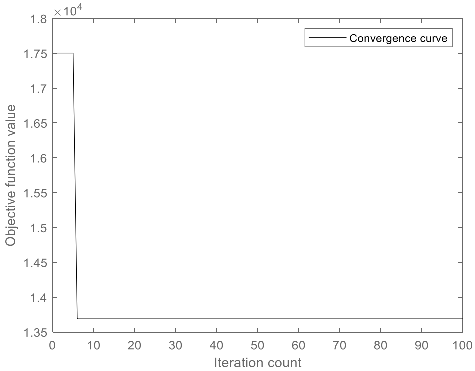

Fig. 4 - Convergence diagram of the optimal solution

5. Conclusion

This paper takes urban one-way circular microcirculation bus routes as the research object, focuses on core issues such as the collaborative optimization of route mileage and scale, and improvement of solution efficiency in route planning, constructs an objective function model of comprehensive mileage minimization with penalty values, designs a hybrid solution strategy combining ant colony algorithm and Dijkstra algorithm, and verifies the effectiveness and feasibility of the model and algorithm through actual numerical examples. The core innovations of this paper are reflected in two aspects: model construction and solution strategy design, as follows:

(1) A flexible constraint optimization model based on penalty values is constructed to achieve collaborative optimization of route mileage and scale. Breaking through the limitations of traditional planning models that set minimum trip distance and minimum number of stops visited constraints, a single-route mileage penalty value λ is introduced into the objective function. By imposing a penalty value on newly added routes, the growth in the number of routes is suppressed, achieving a dynamic collaborative balance between total route mileage and service scale. The model retains only the maximum trip hard constraint and abandons the minimum trip constraint, effectively avoiding the problems of no solution or unreasonable route detours caused by improper

setting of minimum trip parameters. At the same time, the removal of the minimum number of stops visited constraint avoids conflicts with the maximum trip constraint, greatly enhancing the model's adaptability and generalization capability for different road network scenarios.

(2) A hybrid ant colony-Dijkstra solution framework with dynamic origin-destination stops is designed to adapt to dynamic road network topology. Abandoning the fixed origin-destination stop design of traditional algorithms, this paper constructs a probabilistic model based on heuristic functions to achieve dynamic and intelligent selection of bus origin-destination stops, better aligning with the actual operational needs of microcirculation bus. The ant colony algorithm and Dijkstra algorithm are deeply integrated, utilizing the ant colony algorithm for global search of target stops and the Dijkstra algorithm for solving the shortest path between stops based on the current adjacency matrix, balancing the global optimization capability and local path optimization capability of the algorithm.

(3) A dynamic adjacency matrix and along-route node elimination strategy are proposed, significantly improving algorithm solution efficiency. A one-step-updated dynamic adjacency matrix mechanism is designed; each time an ant completes a stop selection, the adjacency matrix is immediately updated and visited nodes are eliminated to prevent repeated node visits. Meanwhile, unvisited bus stops along the shortest path from the current stop to the selected stop are also marked as visited and removed from the set of stops to be visited, reducing the scale of stops to be visited and reducing computational load at the source.

(4) A look-ahead judgment mechanism for trip constraints is introduced to eliminate infeasible trip solutions at the source and ensure route trip feasibility. When the ant selects a target stop at each step, a look-ahead judgment logic is introduced to calculate in real time the current accumulated mileage, the shortest distance from the current stop to the target stop based on the current adjacency matrix, and the actual shortest distance from the target stop to the origin-destination stop calculated based on the updated adjacency matrix after the target stop has been selected and all visited nodes between the current stop and the target stop have been removed. The sum of these three values is then verified against the maximum trip constraint. Path construction continues only when the constraint is satisfied; otherwise, the ant immediately returns from the current stop to the origin-destination stop to form a closed route, ensuring that all generated initial paths strictly satisfy the maximum trip constraint, fundamentally reducing the generation of infeasible solutions, reducing computational redundancy, and improving algorithm stability and solution efficiency.

The research results of this paper provide new ideas and methods for the planning of urban circular microcirculation bus routes. The flexible constraint optimization model and efficient hybrid solution algorithm constructed effectively solve practical planning challenges such as the collaborative optimization of route mileage and scale, and infeasible solution redundancy. The relevant methods and conclusions possess certain engineering application value and significance for promotion.

References

1. Yang, Y., Chen, X., Wang, J., Dong, Y., Qie, K., & Yuan, Z. (2026). Enhancing vision-based traffic crash detection performance consistency across day-night scenes: A depth-aware and domain-adaptive network. Accident; analysis and prevention (Vols. 228, p. 108405).
2. Yang, Y., Huang, H., Li, G., Han, B., Yuan, Z., & Ma, H. (2025). A systematic review of resilience assessment and enhancement of urban integrated transportation networks. Journal of Transport Geography.
3. Du, B., Qiao, Y., Zhao, J., Sun, L., Lv, W., & Huang, R. (2019). Urban Micro-Circulation Bus Planning Based on Temporal and Spatial Travel Demand. 2019 IEEE SmartWorld, Ubiquitous Intelligence & Computing, Advanced & Trusted Computing, Scalable Computing & Communications, Cloud & Big Data Computing, Internet of People and Smart City Innovation (SmartWorld/SCALCOM/UIC/ATC/CBDCom/IOP/SCI).
4. Chen, Y., Lai, Y., Easa, S. M., & Wang, S. (2024). Optimization Model and Solution Design and Coordinated Schedules Considering the Impact of Shared Bicycles. Journal of Transportation Engineering, Part A: Systems.
5. Hu, H., Goswami, R., Jiang, H., & Samaranayake, S. (2025). Optimal Micro-Transit Zoning Via Clique Generation and Integer Programming. 2025 IEEE 28th International Conference on Intelligent Transportation Systems (ITSC).
6. Jiang, T., Jia, X., & Li, J. (2025). Micro-Circulation Bus Route Area Identification under the TOD Concept. SAE Technical Paper Series.
7. Li, R., & Li, G. (2020). Research on Bus Route Layout of Regional Microcirculation Based on Large Data. CICTP 2020 (pp. 2838-2850).
8. Cao, Y., Jiang, D., & Wang, S. (2022). Optimization for Feeder Bus Route Model Design with Station Transfer. Sustainability.
9. Xiong, J., Guan, W., Song, L., Huang, A., & Shao, C. (2013). Optimal Routing Design of a Community Shuttle for Metro Stations. Journal of Transportation Engineering-asce (Vols. 139, pp. 1211-1223).
10. Zhang, B., Zhong, Z., Zhou, X., Qu, Y., & Li, F. (2023). Optimization Model and Solution Algorithm for Rural Customized Bus Route Operation under Multiple Constraints. Sustainability.
11. Yanık, S., & Yilmaz, S. (2022). Optimal design of a bus route with short-turn services. Public Transport (Vols. 15, pp. 169-197).
12. Zhu, Z., Guo, X., Zeng, J., & Zhang, S. (2017). Route Design Model of Feeder Bus Service for Urban Rail Transit Stations. Mathematical Problems in Engineering (Vols. 2017, pp. 1-6).
13. Guo, X., Song, R., He, S., Hao, S., Zheng, L., & Jin, G. (2019). A Multi-Objective Programming Approach to Design Feeder Bus Route for High-Speed Rail Stations. Symmetry (Vols. 11, p. 514).
14. Hu, M., Szeto, W. Y., & Wang, Y. (2024). Cooperative design of feeder bus and bike-sharing systems. Transportmetrica A: Transport Science (Vols. 21).
15. Gao, X., Liu, S., Jiang, S., Yu, D. Z., Peng, Y., Ma, X., & Lin, W. (2024). Optimizing the Three-Dimensional Multi-Objective of Feeder Bus Routes Considering the Timetable. Mathematics.
16. Wang, L.; Yang, M.; Li, X.; Qian, Y. (2025). A Microcirculation Optimization Model for Public Transportation Networks in Low-Density Areas Considering Equity—A Case of Lanzhou. Sustainability 2025, 17, 5679.
17. Su, Q., Luo, X., Zhang, Y., Bao, H., Wang, H. (2026). Multi-objective Route Planning for Shared Micro-circulation Transit in Future Urban Mobility: A Community-Oriented Approach. In: Razminia, A., Nguyen, D.H., Holderbaum, W. (eds) Intelligent Technology for Future Transportation. ITFT 2025. Communications in Computer and Information Science, vol 2876. Springer, Cham.

18. Fang, Q., Li, Z., Wang, Z., Wu, J., Yu, H., & Leng, M. (2019). Research on Planning and Model Design of Micro-cycle Bus Line Based on Metro Station Connection. Journal of Management Science & Engineering Research, 2(1), 13–19.
19. Chen, Q., Shi, F. (2012). Model for Microcirculation Transportation Network Design. Mathematical Problems in Engineering, 2012, 379867, 11 pages, 2012.